Computational Mathematics

Linear Interpolation

**Linear interpolation:**

The simplest form of interpolation is to approximate two data points by a straight line. Suppose we are given two points. These two points can be connected linearly as shown fig. below: using the concept of similar triangles, we show that:

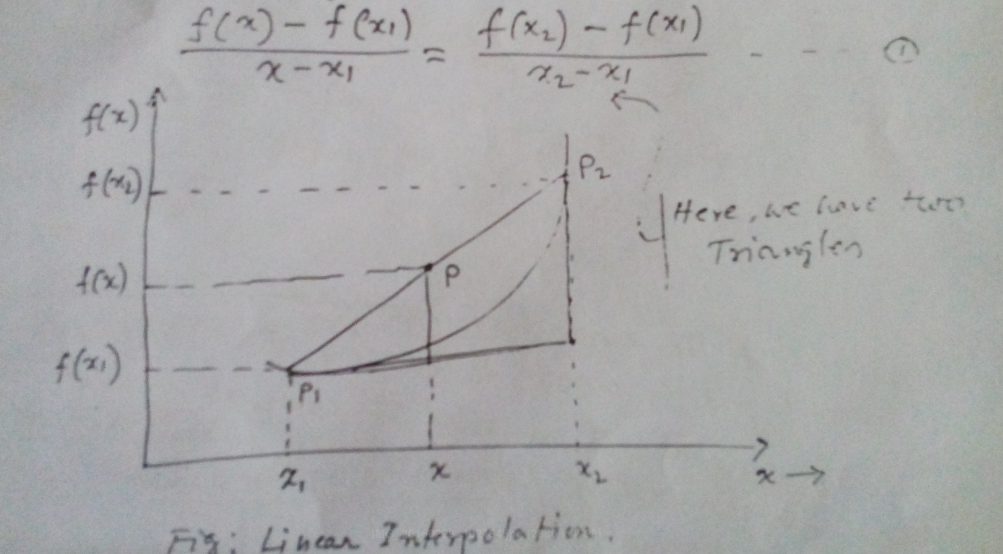


Fig: Linear Interpolation

From (1) we get:

Which is known as Linear Interpolation formula.

**Ex-1:** The table below gives square roots for integers.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 2.5 | 3 | 4 | 5 |
| f(x) | 1 | 1.4142 | 1.5811 | 1.7321 | 2 | 2.2361 |

**Solution:**

The given value of 2.5 lies between the points 2 and 3. Therefore:

Then by the help of linear interpolation formula (2) get:

The correct answer is 1.5811.

The difference is due to the use of a linear model to an interpolation.

Cheek the results to in different interval:

Say though we know must lies between 2 and 3.

Then by (2) we can write:

Error: 1.5811-1.5607=0.0204

Notice that the error has increased from before i.e. 0.0079 to 0.0204

Integral, the smaller the interval between the interpolating data points. The better will be the approximation.

**Ex-2:** Table below gives values of square of integers:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 3.25 | 4 | 5 |
|  | 1 | 4 | 9 | 10.5625 | 16 | 25 |

Using the linear interpolation formula estimate the square root of 3.25

1. Using the points 3 and 4
2. Using the points 2 and 4

[Compare and comment on the results.]

**Ex-3:** Find the linear interpolation polynomial for each of the following pairs of points:

1. (0,1) and (1,3)
2. (-2,3) and (7,12)

[Matlab]

**Other polynomial Form/Interpolation**

1. **Power Form:**
2. **Shifted Form:**
3. **Newton Form:**
4. **Linear Interpolation:**
5. **Newton-Gregory:**

(5.1) Forward Interpolation with equal intervals:

Useful for interpolating the values of f(x) near the beginning of the set of given values.

(5.2) Backward Interpolation with equal interval:

Where are the values of y for the (n+1) equidistant values of

1. **Divided Difference:**

Let the arguments/points to the correspond values of of the polynomial f(x); not necessarily equal spaced. Then the generalized formula for:

(6.1) First divided difference:

Then

………………………………………….

………………………………………….

(6.2) Second divided difference:

Then,

(6.3) Nth Divided Difference:

1. **Newton Divided Difference Formula for unequal interval:**
2. **Lagrange’s Interpolation formula for unequal Intervals:**

**Ex-1:** Newton’s Forward and Backward (both):

Find the value of y at x=21 and x=28 from the following data:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X= | 20 | 23 | 26 | 29 |
| Y= | 0.342 | 0.3907 | 0.4384 | 0.4848 |

**Solution:** First we construct the difference table for the given data as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | y | y | y | y |
| 20 | 0.342 |  |  |  |
|  |  | 0.0487 |  |  |
| 23 | 0.3907 |  | -0.0010 |  |
|  |  | 0.0477 |  | -0.0003 |
| 26 | 0.4384 |  | -0.0013 |  |
|  |  | 0.0464 |  |  |
| 29 | 0.4848 |  |  |  |

1. We have to find y(21):

Here,

Since x=21 is nearer to the beginning of the table. So, we use Newton’s forward interpolation.

i.e.

1. Again we have to find y(28):

Since, x=28 is nearer to end value. So, we use Newton’s Backward Interpolation formula. Here

Therefore the value of y at x=21 and x=28 are 0.3583 and 0.46946 respectively. (**Ans.**)

**Ex-2:** The population of a town in the decimal census was as given below. Estimate the population for the year 1895.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year (x) | 1891 | 1901 | 1911 | 1921 | 1931 |
| Population (y) | 46 | 66 | 81 | 93 | 101 |

**Solution:** Construct the difference table. We have to find y(1895). Since x=1895 is near to the beginning of the table/data. So, we use Newton’s forward interpolation formula.

Here,

**Ex-3:** The following table gives the population of a town during the last six censuses. Estimate using any suitable interpolation formula, the increase in the population during the period from year 1996 to 1998.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Year (x) | 1961 | 1971 | 1981 | 1991 | 2001 | 2011 |
| Population (th) | 12 | 15 | 20 | 27 | 39 | 52 |

**Solution:** Construct the difference table. Use Newton’s backward interpolation formula.

Here,

Then,

**Ex-4:** Find the annual premium at the age of 46 and 63 from the following table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age (x) | 45 | 50 | 55 | 60 | 65 |
| Premium (y) | 114.84 | 96.16 | 83.32 | 74.48 | 68.48 |

1. Forward ⇾
2. Backward ⇾

**Ex-5:** Marks obtained by the students in an examination are given below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Marks (x) | 45 | 50 | 55 | 60 | 65 |
| No of students (y) | 114.84 | 96.16 | 83.32 | 74.48 | 68.48 |

Estimate the number of students who obtained less than

1. 42 marks
2. 70 marks

**Solution:** First we prepare the cumulative frequency table as below:

|  |  |
| --- | --- |
| Marks Obtained | No of students |
| <20 | 41 |
| <40 | 41+62=103 |
| <60 | 103+65=168 |
| <80 | 168+50=228 |
| <100 | 218+17=235 |

Difference table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Marks (x) | y | y | y | y | y |
| 20 | 41 |  |  |  |  |
|  |  | 62 |  |  |  |
| 40 | 103 |  | 3 |  |  |
|  |  | 65 |  | -18 |  |
| 60 | 168 |  | -15 |  | 0 |
|  |  | 50 |  | -18 |  |
| 80 | 218 |  | -33 |  |  |
|  |  | 17 |  |  |  |
| 100 | 235 |  |  |  |  |

1. For 42:

Here,

We use Newton’s forward interpolation.

i.e.

The number of students who obtained less than 42 marks=110 (**Ans.**)

1. For 70:

We use Newton’s Backward Interpolation formula. Here

The number of students who obtained less than 70 marks =196 (**Ans.**)

**Ex-6:** Marks obtained by the students in an examination are given below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Marks (x) | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| No of students (y) | 31 | 42 | 51 | 35 | 31 |

Estimate the number of students who obtained:

1. Less than 45 marks
2. Greater than 45 marks
3. Between 40 and 45 marks

**Ex-7:** Find the polynomial f(x) passes through the points:

**Solution:** we construct the difference table for the given data is as below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | y | y | y | y |
| 0 | 1 |  |  |  |
|  |  | 2 |  |  |
| 1 | 3 |  | 2 |  |
|  |  | 4 |  | 0 |
| 2 | 7 |  | 2 |  |
|  |  | 6 |  |  |
| 3 | 13 |  |  |  |

Here,

So, by Newton’s forward interpolation formula: we get:

**Ex-8:** Using the following table find the Function f(x):

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X: | 0 | 1 | 2 | 3 | 4 |
| f(x): | 3 | 6 | 11 | 18 | 27 |